An Upper Bound on Multihop Transmission Capacity With Dynamic Routing Selection

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***Abstract*—This paper develops upper bounds on the end-to-end transmission capacity of multihop wireless networks. Potential source–destination paths are dynamically selected from a pool of randomly located relays, from which a closed-form lower bound on the outage probability is derived in terms of the expected number of potential paths. This is in turn used to provide an upper bound on the number of successful transmissions that can occur per unit area, which is known as the transmission capacity. The upper bound results from assuming independence among the potential paths, and can be viewed as the maximum diversity case. A useful aspect of the upper bound is its simple form for an arbitrary-sized network, which allows insights into how the number of hops and other network parameters affect spatial throughput in the nonasymptotic regime. The outage probability analysis is then extended to account for retransmissions with a maximum number of allowed attempts. In contrast to prevailing wisdom, we show that predetermined routing (such as nearest neighbor) is suboptimal, since more hops are not useful once the network is interference-limited. Our results also make clear that randomness in the location of relay sets and dynamically varying channel states is helpful in obtaining higher aggregate throughput, and that dynamic route selection should be used to exploit path diversity.**

***Index Terms*—Multihop routing, outage probability, signal-to-interference-plus-noise ratio (SINR), stochastic geometry, transmission capacity.**

I. Introduction

In a distributed wireless network with random node locations, determining the precise network capacity is a longstanding open problem that includes many other simpler open problems as special cases [1]. Therefore, suboptimal analytical approaches that provide insights into the achievable throughput and inform improved protocol design are well motivated, even if they fall short of strict upper bounds. Multihop routing is generally considered necessary in large wireless networks, both to ensure connectivity and to improve throughput, but it is typically not well optimized, nor are its performance limits in general models known. In this paper, we explore optimal multihop strategies by considering dynamic path selection. Predetermined routing strategies such as nearest-neighbor routing, although they may perform fairly well on average, are generally not optimal for a given network state (which includes node positions and all the channels among them). In fact, a randomly deployed set of potential relays with time-varying fading channels may allow a large gain by providing more potential routes. In this paper, we are interested in how the inherent randomness in the network can be better harvested to improve the end-to-end success probability and, hence, throughput over more static approaches.

We assume that node locations are a realization of homogeneous Poisson process in order to investigate the throughput statistically. This model provides a tractable way to characterize how the end-to-end success probability and throughput varies over different multihop routing strategies. We aim to study how multihop routing with the assistance of a pool of randomly deployed relays impacts the throughput scaling in a nonasymptotic regime, i.e., for networks of finite population and area, with the goal of finding how much the average throughput can be increased under quality of service (QoS) constraints. Considering uncoordinated routing selection, we aim at determining the fundamental limits for a general class of routing strategies instead of predetermined selection. It can be expected that the diversity gain resulting from the randomness and dynamic channels, potentially, provides significant throughput improvement.

A. Related Work and Motivation

The best known metric for studying end-to-end network capacity is the transport capacity [2]–[4]. This framework pioneered many notable studies on the limiting scaling behavior of ad hoc networks with the number of nodes n by showing that the maximum transport capacity scales as $\Theta\left(\sqrt{n}\right)$ in arbitrary networks [2]. The feasibility of this throughput scaling has also been shown in random networks by relaying all information via crossing paths constructed through the network [5]. Several other researchers have extended this framework to more general operating regimes, e.g., [6] and [7]. Their findings have shown that nearest-neighbor multihop routing is order optimal in the power-limited regime, while hopping across clusters with distributed multiple-input and multiple-output communication can achieve order-optimal throughput in bandwidth-limited and power-inefficient regimes. However, most of these results are shown and proven for asymptotically large networks, which may not accurately describe nonasymptotic conditions. Moreover, scaling laws do not provide much information on how other network parameters imposed by a specific transmission strategy affect the throughput.

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If node locations are modeled as a homogeneous Poisson point process (HPPP), a number of results can be applied from stochastic geometry, e.g., [8] and [9], in particular to compute outage probability relative to a signal-to-interference-plus-noise ratio (SINR) threshold. These expressions can be inverted to give the maximum transmit intensity at a specified outage probability, which yields the transmission capacity of the network [10]. This framework provides the maximum number of successful transmission the network can support while simultaneously meeting a network-wide QoS requirement. This framework allows closed-form expressions of achievable throughput to be derived in nonasymptotic regimes, which are useful in examining how various communication technique, channel models, and design parameters affect the aggregate throughput, e.g., [11]–[18]; see [19] for a summary. While the transmission capacity can often be expressed in closed form without resorting to asymptotics, it is a single hop or "snapshot" metric. Recent work [20], [21] began to investigate the throughput scaling with two-hop opportunistic relay selection under different channel gain distribution and relay deployment. However, more general multihop capacity has not proven tractable.

If several other strong assumptions are made, e.g., that all relays are placed equidistant on a straight line and all outages are independent, then closed-form multihop transmission capacity can be derived [22]. Stamatiou *et al.* [23] also investigated multihop routing in a Poisson spatial model, whose focus is to characterize the end-to-end delay and stability, again based on predetermined routes. Other recent works analyzing the throughput of multihop networks using stochastic geometric tools include [24], which extended [22] to nonslotted ALOHA and [25], which also adopted a similar framework to [25] to study the throughput-delay-reliability tradeoff with an ARQ protocol, and did not require all hops to be equidistant. However, all of these used predetermined routing selection. In fact, the outage of a predetermined route does not preclude the possibility of successful communication over other routes. Separately, multihop capacity has also been studied in a line network without explicitly considering additional interference [26], [27]. This approach is helpful in comparing the impact of additional hops in bandwidth and power-limited networks, but fails to account for the interference inherent in a large wireless network.

In addition, the aforementioned diversity gain from dynamic relay selection has been utilized for opportunistic routing [28], [29], so any node that overhears packets can participate in forwarding. The work in [29] appeared to be the first investigation of the capacity improvement from opportunistic routing compared with predetermined routing in a Poisson field. However, the performance gain shown in [29] is based on simulation without an exact mathematical derivation. Different random hop selection strategies have also been studied and compared [30], [31] without giving tractable throughput bounds. Hence, characterizing the available diversity gain is worth investigating. In this paper, we will explicitly show that since a pool of randomly located relays with varying channels provides more potential routes, more randomness is preferable.

B. Contributions and Organization

Instead of predetermined routing, dynamic route selection from random relay sets under varying channel states is investigated in this paper. The main contributions are summarized as follows.

1) We provide a lower bound on the end-to-end outage probability (Theorem 1), which can be expressed as an exponential function with respect to the expected number of potential paths. This result implies that higher throughput can be achieved when the correlation among the states of different hops is low and hence randomness and opportunism is high.
2) We further derive in closed form the expected number of $\mathcal{S}-\mathcal{D}$ routes that can complete forwarding both for single transmission in each hop and for two different retransmission strategies that are subject to constraints on the number of allowed attempts, given in Lemma 1. The basic idea is to map all relay combinations to a higher dimensional space and focus on the level set with respect to the success probability function.
3) A closed-form upper bound on transmission capacity as a function of outage constraint ϵ and the number of relays m for a general class of multihop routing strategies are given in Corollary 2, which follows from Lemma 1 and is the main technical result in this paper. These closed-form results assume a general exponential form of success probability, which includes most commonly used channel models as special cases, including path loss, path loss with Rayleigh fading, and path loss with Nakagami fading.
   The aforementioned results show that in networks with uncoordinated routing, an "ideal" diversity gain arising from independent statistics of different paths allows the throughput to exhibit near linear scaling in the number of relays m as long as the density of relay nodes exceeds the threshold imposed by the outage constraint ϵ. This diversity gain requires strong "incoherence" among different paths, which would presumably degrade for large m since longer routes are more likely to be correlated or share common links. Unlike the single-hop scenario where network throughput must decrease about linearly as the output constraint ϵ is tightened, the multihop capacity bound is less sensitive to ϵ especially for large m.
4) Finally, we briefly show that all predetermined routing strategies with no central coordination and without further information like channel state information may fail to outperform single-hop transmission in an interference-limited network because of the large increase in interference. Hence, exploiting randomness is important for multihopping to be viable in networks of finite size.

The rest of this paper is organized as follows. In Section II, we first define the end-to-end metric that quantifies the network-wide throughput, and then state the key assumptions for the analysis, as well as the channel models and their associated general form of per-hop success probability. We then develop and derive lower bounds for end-to-end outage probability for general scenarios in Section III-A. Specifically, this provides a closed-form lower bound if the channel model allows the per-hop success probability to be expressed in exponential form, which is developed in Section III-B. This in turn results in an upper bound for the multihop transmission capacity in Section III-C. Implications and interpretations of the results are provided in Section IV.

II. Models and Preliminaries

A. Models and Assumptions

We assume that the locations of all sources are a realization of an HPPP Ξ_t of intensity λ_t, and a set of relays are also randomly deployed in the plane with homogeneous Poisson distribution independent of Ξ_t. We consider a fixed-portion model, i.e., the relay set is of spatial density $\frac{1-\gamma}{\gamma}\lambda_t$, where $\gamma \in (0,1)$ is assumed to be a fixed constant. In other words, if the locations of all wireless nodes are assumed to be an HPPP Ξ with intensity λ, then the set of active transmitters is of intensity $\lambda_t = \gamma\lambda$. The destination node is assumed to be a distance R away from its associated source node in a random direction, and is not a part of the HPPP. Suppose transmission rate $b \approx \frac{1}{2}\log(1+\beta)$ is required for successful transmission, where β is, therefore, the required SINR. Denoting ϵ as the target outage probability relative to β, the transmission capacity [10] in an uncoordinated single-hop setting is defined as

$$T(\epsilon) = (1-\epsilon) \max_{\mathbb{P}(\mathrm{SINR}<\beta)\le\epsilon} \lambda_t \tag{1}$$

which is the maximum expected throughput per unit area. Since b is simply a constant function of β, we ignore it for simplicity.

Now suppose each session uses k transmissions with the assistance of the relay set. These k attempts can be performed in an arbitrary k orthogonal slots, i.e., the unit time slot can be divided into k equal subslots and the source and relays take turns transmitting in these subslots: only one transmitter per route is active at a time. The contention density is still of density λ_t, but each packet is transmitted k times. Therefore, the multihop transmission capacity metric should be modified to be

$$T_m(\epsilon) = (1-\epsilon) \max_{\mathbb{P}(\mathrm{SIR}<\beta)\le\epsilon} \frac{\lambda_t}{k} \tag{2}$$

since each hop requires a time slot, so the overall throughput must be normalized by k. It should be noted that although one can "pipeline" by simultaneously transmitting different packets on different hops, this does not change the transmission capacity since the contention density simply becomes k times larger. Similar analysis can be applied in quantifying the transmission capacity with this intraroute spatial reuse but leads to the same result. When no retransmissions are allowed, we have $k = m+1$ with m relays; if we consider M total attempts (including retransmissions) for any single session, then $k = M$.

Slivnyak's theorem [32] states that an entire homogeneous network can be characterized by a typical single transmission. Conditioning on a typical pair, the spatial point process is still homogeneous with the same statistics. Suppose that all transmitters employ equal amounts of power, and the network is interference-limited, i.e., noise power is negligible compared to interference power. Relays can be selected from all nodes in the feasible region. In this paper, we consider the effects of both path loss and fading. For point-to-point transmission from node i to node j at a distance r_{ij}, the requirement for successful reception

TABLE I
Summary of Notation and Parameters

α	path loss exponent
β	SINR requirement for successful reception per hop
ϵ	outage probability constraint
R	$\mathcal{S}-\mathcal{D}$ transmit distance
λ	contention density of all potential transmitters, intensity of Ξ
λ_t	contention density of active transmitters in any subslot, intensity of Ξ_t
γ	the portion of active transmitters, $\lambda_t = \lambda\gamma$
r_{ij}	Euclidean distance from node i to node j
K, G	general parameters in exponential form single-hop success probability, $g_0(r_{ij},\lambda_t) = G\exp\left(-\lambda_t K r_{ij}^2\right)$
$p_{\mathrm{out}}^{(m)}$	end-to-end outage probability employing m relays without retransmissions
Z_m	the corresponding location vector in $\mathscr{R}^{2m}$
$g_0(r_{ij},\lambda_t)$	single hop success probability
$g_m(Z_m,\lambda_t)$	the end-to-end ($m+1$ hops) success probability with the relay set Z_m
K, G	general parameters in exponential form single-hop success probability, $g_0(r_{ij},\lambda_t) = G\exp\left(-\lambda_t K r_{ij}^2\right)$
$v_{2m}(B)$	Lebesgue measure of B
$d_m(Z_m)$	sum of squared distance of $m+1$ hops with the relay set Z_m
D_m	distance constraint, $d_m(Z_m) \le D_m$
$T_m(\epsilon)$	$m+1$ hop transmission capacity
Λ	$\Lambda := \lambda\gamma K$
κ	$\kappa := G\pi(1-\gamma)/\gamma K$

in this hop is expressed in terms of signal-to-interference ratio (SIR) constraint as

$$\mathrm{SIR}_{ij} = \frac{\|h_{ij}\|^2 r_{ij}^{-\alpha}}{\sum_{k\ne i}\|h_{kj}\|^2 r_{kj}^{-\alpha}} \ge \beta \tag{3}$$

where α denotes the path loss exponent, and h_{ij} is the fading factor experienced by the path from i to j. Distinct links are assumed to experience i.i.d. fading, which is typically reasonable. Notation is summarized in Table I.

B. Per-Hop Success Probability

A Poisson node distribution often results in an exact or approximate exponential form for per-hop successful probability. That is, given that the packet is transmitted from node i to next hop receiver j over distance r_{ij} and contention density λ_t, the probability that the received SIR stays above the target β can be expressed as

$$\mathbb{P}\left(\mathrm{SINR}_{ij} > \beta\right) \stackrel{\triangle}{=} g_0(r_{ij},\lambda_t) = G\exp\left(-\lambda_t K r_{ij}^2\right) \tag{4}$$

where G and K depend on the specific channel models and are independent of r_{ij} and λ_t. This holds for several commonly used channel models, including Rayleigh fading with path loss, Nakagami fading with path loss, and path loss without fading, as we briefly show in this section before using the general form in the remainder of this paper.

1) Rayleigh Fading: Baccelli *et al.* [8] showed under Rayleigh fading that

$$g_0(r_{ij},\lambda_t) = \exp\left(-\lambda_t r_{ij}^2 \beta^{2/\alpha} C(\alpha)\right) \tag{5}$$

where $C(\alpha) = 2\pi\Gamma(\frac{2}{\alpha})\Gamma(1-\frac{2}{\alpha})/\alpha$ with $\Gamma(z) = \int_0^\infty t^{z-1}\exp(-t)dt$ being the Gamma function. Hence, the coefficients under Rayleigh fading can be given as

$$K_{\rm RF} = \beta^{\frac{2}{\alpha}} C(\alpha); \quad G_{\rm RF} = 1. \tag{6}$$

2) Nakagami Fading: Nakagami fading is a more general fading distribution, whose power distribution can be expressed in terms of fading parameter $m_0 \geq 0.5$ as

$$f_Z(z) = \frac{m_0^{m_0} z^{m_0-1}}{\Gamma(m_0)} \exp(-m_0 z). \tag{7}$$

Recent work [15] has suggested a way to study the outage probability by looking at the Laplacian transform. In both low-outage and high-outage regimes, the success probability under Nakagami fading can be expressed as an exponential function with the following coefficients:

$$\text{low-outage regime: } K_{\rm NF} = \Omega_{m_0}\beta^{\frac{2}{\alpha}}, \; G_{\rm NF} = 1$$
$$\text{high-outage regime: } K_{\rm NF} = \Omega_{m_0}\beta^{\frac{2}{\alpha}}$$
$$G = 1 + \sum_{k=1}^{m_0-1}\sum_{l=1}^{k} \frac{l!}{k!}\left(\frac{-2}{\alpha}\right)^l \Upsilon_{k,l}.$$

Both the derivation of these coefficients and the definition of Ω_{m_0} and $\Upsilon_{k,l}$ can be found in Appendix A. These two regimes are typical in practical systems.

3) Path Loss Model (Nonfading): The exact closed-form formula of the success probability with only path loss is unknown. One approach is letting $m_0 \to \infty$ for Nakagami fading, which converges to a path-loss-only model. However, lower and upper bounds follow an exponential form. For example, partitioning the set of interferers into dominating and nondominating nodes, an upper bound can be obtained as $g_0^{\rm ub}(r_{ij}, \lambda_t) = \exp(-\lambda_t \pi \beta^{\frac{2}{\alpha}} r_{ij}^2)$. The authors in [10] have also shown an upper bound on the transmission capacity that is $\frac{\alpha}{\alpha-1}$ times the lower bound for small ϵ, and has illustrated by simulation the tightness of these bounds. This suggests that there exists some constant $K_{\rm PL}$ such that

$$g_0(r_{ij}, \lambda_t) = \exp\left(-\lambda_t K_{\rm PL} r_{ij}^2\right) \tag{8}$$

where

$$\pi\beta^{\frac{2}{\alpha}} \leq K_{\rm PL} \leq \frac{\alpha}{\alpha-1}\pi\beta^{\frac{2}{\alpha}}, \quad G_{\rm PL} = 1. \tag{9}$$

III. Main Results

In this section, we first address the end-to-end outage probability for general channel models, which builds a connection between outage probability and expected number of potential paths. Next, if channel models allow the per-hop success probability to be expressed in an exponential form, closed-form lower bounds can be derived. This in turn will provide a closed-form upper bound for multihop transmission capacity.

A. Outage Probability Analysis for General Per-Hop Success Probability

Suppose that m relays are employed by a typical source-destination ($\mathcal{S}$–$\mathcal{D}$) pair. Since all $\mathcal{S}$–$\mathcal{D}$ pairs are stochastically equivalent, we can investigate the performance by looking at a typical $\mathcal{S}$–$\mathcal{D}$ pair. We will build a connection between the outage probability and the expected number of relay sets that can connect the source and destination. Suppose that there is a transmission pair with source and destination located at $(-R/2, 0)$ and $(R/2, 0)$, respectively. With the $i{\rm th}(1 \leq i \leq m)$ relay located at (x_i, y_i), let $Z_m = (x_1, y_1, \ldots, x_m, y_m)$ denote the location vector of this specific relay set. From Slivnyak's theorem, conditional on a typical transmission pair or finite number of nodes, the remaining point process is still homogeneous Poisson process with the same spatial density (we ignore a finite number of singular points here). Therefore, all relay combinations form a homogeneous point process in a $2m$-dimensional space $\mathcal{R}^{2m}$, as illustrated in Fig. 1. The effective spatial density is $\tilde{\lambda} = (1-\gamma)\lambda$, which characterizes the density of the pool of nodes that have not been designated for specific transmissions. Assume that each relay combination Z_m can successfully assist in communication between the $\mathcal{S}$–$\mathcal{D}$ pair with probability $g_m(Z_m, \lambda_t)$. If we call a relay set that can successfully complete forwarding in a given realization of the spatial process a potential relay set, then the expected number of potential relay sets in a hypercube B, denoted by N_B, can be expressed as

$$\begin{aligned} &\lim_{v_{2m}(B)\to 0} \mathbb{E}(N_B) \\ &= \lim_{v_{2m}(B)\to 0} \mathbb{E}\left(\sum_{Z_m \in B} \mathbb{I}\left(Z_m \text{is a potential relay set}\right)\right) \\ &= \tilde{\lambda}^m g_m(Z_m, \lambda_t) v_{2m}(B) \end{aligned} \tag{10}$$

where $v_{2m}(B)$ denotes the Lebesgue measure of B and $\mathbb{I}(\cdot)$ denotes indicator function. Let the random variable N_m be the number of relay sets that can complete forwarding using $m+1$ hops. Then $\mathbb{E}(N_m)$ is the expected number of different routes that can successfully forward the packets for an $\mathcal{S}$–$\mathcal{D}$ pair. A larger $\mathbb{E}(N_m)$ naturally leads to lower outage, where we formalize in the following theorem.

Theorem 1: Assume that all end-to-end transmissions are achieved via $m+1$ hops with m relays. The end-to-end outage probability for any $\mathcal{S}$–$\mathcal{D}$ pair $p_{\rm out}^{(m)}$ can be lower bounded as

$$p_{\rm out}^{(m)} \geq \exp(-\mathbb{E}(N_m)). \tag{11}$$

Proof: The key idea is to view the outage event as the intersection of a set of *decreasing events*. The basic properties of decreasing events suggest a lower bound by treating all these events as mutually independent. See Appendix B for complete proof. ■

The lower bound can only be approached when the pool of potential relay sets form a Poisson point process in a corresponding $2m$-dimensional space (detailed in Appendix B), i.e., all potential relay combinations independent from each other. In practice, route selection for different source–destination pairs are not independent, so the derived lower bound is not obtained by realistic routing strategies. This result, however, indicates that low correlation among different routes can reduce the outage probability in essence by enhancing diversity.

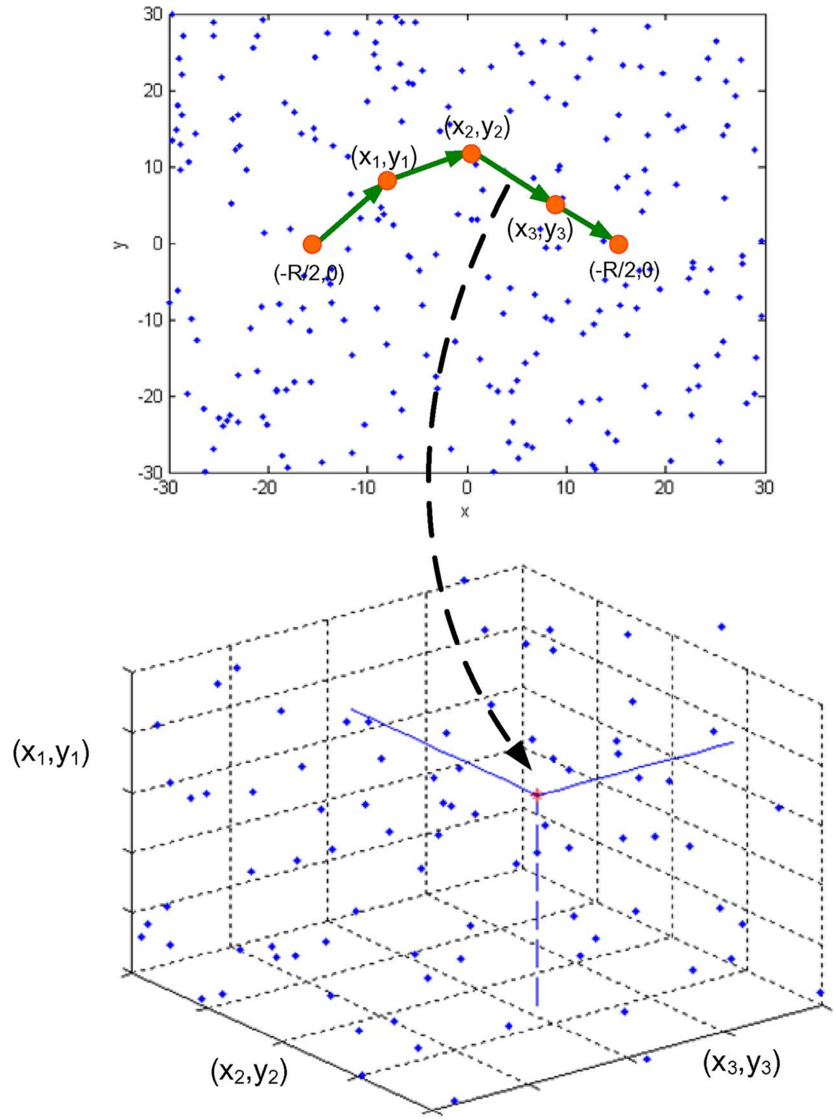

Fig. 1. In the left plot, the $\mathcal{S}$–$\mathcal{D}$ pair use three relays to assist the transmission, which can be matched to a point in high-dimensional space as plotted in the right plot. In fact, a precise plot requires drawing on 6-D space with each relay accounting for two dimensions, but the right plot may help explain the mapping intuitively.

We conjecture that this bound is tight and reasonable for small m (e.g., the bound is exact for single relay case) but may be loose for large m. This is because for a fixed pool of relays, the correlation among different routes increases when the number of relays m grows, i.e., for large m, many routes are likely to share at least one link. Also, the bound may become loose for low-outage regime (e.g., $\epsilon < 10^{-3}$), because the outage bound is an exponential function of $\mathbb{E}\left(N_m\right)$, where even constant factor difference may result in an exponential gap. The advantage of this bound is to allow us to see how the maximum allowable throughput scales with fixed normal outage constraint in the nonasymptotic regime, which will be shown later. In fact, some constant correction factor can also be applied based on some simulation results without changing the scaling of maximum contention density versus outage constraint. The expected number of different routes $\mathbb{E}\left[N_m\right]$ plays an important role, which will be calculated exactly in the following section.

We caution, however, that the key assumption of independent path selection cannot be achieved in practice. Potential routes are virtually coupled and never independent from each other. For instance, failure of finding a potential route for one session typically implies a lower success probability for another session. But when uncoordinated routing selection is employed for all $\mathcal{S}$–$\mathcal{D}$ pairs and when the number of relays is reasonably small, the correlation is mitigated. And this simplication assumption allows a reasonable closed-form bound to be derived.

B. Outage Probability Analysis for Exponential-Form Per-Hop Success Probability

Now we begin to concentrate on the success probability of exponential forms. When no retransmissions are adopted, if a specific route is selected for packet delivering over m relays with hop distances $r_1, r_2, \ldots, r_{m+1}$, respectively, the probability for successful reception can be found as the product of each hop's success probability

$$g_m(r_1, \ldots, r_{m+1}, \lambda_t) \tag{12}$$

$$= \prod_{i=1}^{m+1} g_0(r_i, \lambda_t) = G^{m+1} \exp\left(-\lambda_t K \sum_{i=1}^{m+1} r_i^2\right). \tag{13}$$

Here, we assume independence among the success probability of each hop, which requires sufficient diversity in the interferer locations over time. Although this assumption is not valid in practice since transmissions across subsequent hops are correlated, it allows us to retain tractability and has been shown to be a reasonable approximation in uncoordinated networks (as illustrated in [22]). The temporal correlation may also be mitigated through diversity techniques like frequency hopping.

Conditional on a typical transmission pair with source and destination located at $(-R/2, 0)$ and $(R/2, 0)$, the spatial point process is still an HPPP with the same statistics. In the m relay case with the $i\text{th}(1 \leq i \leq m)$ relay located at (x_i, y_i), let $Z_m = (x_1, y_1, \ldots, x_m, y_m)$ denote the locations of the specific relay set; then, we can define the corresponding distance statistics as

$$d_m(Z_m) \triangleq (x_1 + \frac{R}{2})^2 + \sum_{i=1}^{m-1} (x_i - x_{i+1})^2 + (x_m - \frac{R}{2})^2 + y_1^2 + \sum_{i=1}^{m-1} (y_i - y_{i+1})^2 + y_m^2. \quad (14)$$

This is the sum of squares of hop distances. Hence, the routing success probability for a specific set of relays with location Z_m can be explicitly expressed as

$$g_m(Z_m, \lambda_t) = G^{m+1} \exp(-\lambda_t K d_m(Z_m)). \quad (15)$$

In fact, an arbitrary set of relays will have positive probability for successful forwarding. However, for those relay sets with large $d_m(Z_m)$, the communication process becomes extremely fragile and difficult to maintain due to the low reception probability and large distance. Practical protocols usually attempt to search potential routes inside a locally finite area instead of from the infinite space since the longer routes are very unlikely to be an efficient one. In order to leave the analysis general, we impose a constraint $d_m(Z_m) \leq D_m$ for the m relay case, where $D_m \to \infty$ reverts to the unconstrained distance case. We will later show that a reasonably small constraint D_m is sufficient to achieve an aggregate rate arbitrarily close to the capacity upper bound.

Moreover, since only one transmitter is active at a time along the entire multihop route, in each subslot, each node is used as a relay by other source–destination pairs with probability γ. Therefore, the pool of relays in each hop can be treated as the original point process Ξ with each point being deleted with probability γ. Hence, the location of all relay sets in $\mathscr{R}^{2m}$ can be viewed as a realization of a point process with effective spatial density $\lambda^m(1-\gamma)^m$. This leads to the following lemma.

Lemma 1: Define $\kappa = G\pi(1-\gamma)/\gamma K$ and $\Lambda = \lambda\gamma K$. If all end-to-end transmissions are achieved via $m+1$ hops with m relays, with a constraint $d_m(Z_m) \leq D_m$ ($Z_m \in \mathscr{R}^{2m}$), the expected number of potential relay sets can be computed as

$$\mathbb{E}(N_m) = \frac{G\kappa^m}{m+1} \left\{ \exp\left(-\frac{\Lambda R^2}{m+1}\right) - \exp\left(-\Lambda D_m\right) \cdot \sum_{0}^{m-1} \frac{1}{i!} \left(\Lambda \left(D_m - \frac{R^2}{m+1}\right)\right)^i \right\}. \quad (16)$$

Proof: The key point in the proof is that the isosurface of $d_m(Z_m)$ forms a high-dimensional elliptical surface, which provides a tractable closed-form solution. See Appendix C. ■

This result indicates that a larger m typically provides more diversity, because it provides more possible combinations of different relays, and the dynamically changing channel states provide more opportunities for us to find a potential route. A larger feasible range for route selection D_m also increases the expectation, but since the effect of D_m mainly exhibits as an exponentially vanishing term, it can be expected that a fairly small range is enough to approach the limits. Moreover, this analytic framework can be extended to account for retransmissions in the following two scenarios. First, the best effort retransmission protocol requires that each hop adopts k retransmissions regardless of the results of each transmissions. The following lemma provides more general results for best effort protocols by allowing each hop to adopt a different number of retransmissions. Second, instead of specifying retransmissions for each hop, we bound the maximum number of total allowed attempts to M. Define $\mathbf{1} := (1, \ldots, 1)^T$. The following lemma provides closed-form results for these two scenarios.

Lemma 2: Assume that all end-to-end transmissions are achieved via $m+1$ hops with m relays.

1) In the best effort retransmission setting, if the $i\text{th}(1 \leq i \leq m+1)$ hop is retransmitted k_i times, then the expected number of potential relay sets can be given as

$$\mathbb{E}_{\mathbf{k}}(N_m) = \sum_{\mathbf{n}:\mathbf{1}\preceq\mathbf{n}\preceq\mathbf{k}} \frac{(-1)^{m+1}\pi^m(1-\gamma)^m(-G)^{\sum_{j=1}^{m+1} n_j}}{\gamma^m K^m} \cdot \prod_{i=1}^{m+1} \binom{k_i}{n_i} \frac{m \exp\left(-\frac{\Lambda R^2}{\sum_{j=1}^{m+1} 1/k_j}\right)}{\left(\prod_{i=1}^{m+1} n_i\right)\left(\sum_{i=1}^{m+1} \frac{1}{n_i}\right)}. \quad (17)$$

2) If the $\mathcal{S}$–$\mathcal{D}$ transmission allows M transmissions in total without specifying the number of retransmissions for each hop, then the expected number of potential relay sets can be given as

$$\mathbb{E}_M(N_m) = \sum_{\substack{\mathbf{k}^T\cdot\mathbf{1}\leq M \\ \mathbf{k}\succeq\mathbf{1}}} (-1)^{\mathbf{k}^T\cdot\mathbf{1}} \sum_{\substack{\mathbf{j}^T\cdot\mathbf{1}\leq M \\ \mathbf{j}\succeq\mathbf{k}}} \prod_{l=1}^{m+1} \binom{j_l - 1}{k_l - 1} \mathbb{E}_{\mathbf{k}}(N_m) \quad (18)$$

where $\mathbb{E}_{\mathbf{k}}(N_m)$ is given in (17).

Proof: See Appendix D. ■

Similarly, this lemma is derived by mapping all potential relay sets onto a $2m$-dimensional space and investigating the isosurfaces in that space. The aforementioned results on the expected number of potential relay sets immediately yield the following corollary.

Corollary 1: Assume that all end-to-end transmissions are achieved via $m+1$ hops with m relays.

(1) If only a single transmission is allowed for in each hop, then the outage probability under a constraint $d_m(Z_m)\leq D_m$ ($Z_m\in\mathscr{R}^{2m}$) can be computed as

$$p_{\rm out}^{(m)} \geq \exp\left\{-\frac{G\kappa^m}{m+1}\left\{\exp\left(-\frac{\Lambda R^2}{m+1}\right)-\exp\left(-\Lambda D_m\right)\sum_0^{m-1}\frac{1}{i!}\left(\Lambda\left(D_m-\frac{R^2}{m+1}\right)\right)^i\right\}\right\}. \quad (19)$$

(2) If each hop adopts $k_i (1 \leq i \leq m+1)$ retransmissions, the outage probability $q_{\mathbf{k}}$ can be computed as

$$q_{\mathbf{k}} \geq \exp\left(-\sum_{\mathbf{n}:\mathbf{1}\preceq\mathbf{n}\preceq\mathbf{k}}\frac{(-1)^{m+1}\pi^m(1-\gamma)^m(-G)^{\sum_{j=1}^{m+1}n_j}}{\gamma^m K^m}\cdot\prod_{i=1}^{m+1}\binom{k_i}{n_i}\frac{m\exp\left(-\frac{\Lambda R^2}{\sum_{j=1}^{m+1}1/k_j}\right)}{\left(\prod_{i=1}^{m+1}n_i\right)\left(\sum_{i=1}^{m+1}\frac{1}{n_i}\right)}\right. \quad (20)$$

(3) If the transmission adopts M retransmission in total, the outage probability q_M can be given as

$$q_M \geq \exp\left(-\sum_{\substack{\mathbf{k}^T\cdot\mathbf{1}\leq M\\ \mathbf{k}\succeq\mathbf{1}}}(-1)^{\mathbf{k}^T\cdot\mathbf{1}}\cdot\sum_{\substack{\mathbf{j}^T\cdot\mathbf{1}\leq M\\ \mathbf{j}\succeq\mathbf{k}}}\prod_{l=1}^{m+1}\binom{j_l-1}{k_l-1}\mathbb{E}_{\mathbf{k}}\left(N_m\right)\right. \quad (21)$$

where $\mathbb{E}_{\mathbf{k}}\left(N_m\right)$ is given in (17).

This corollary provides closed-form lower bounds on the end-to-end outage probability. For sufficiently large D_m in the nonretransmission case, the lower bound reduces to

$$p_{\rm out}^{(m)} \geq \exp\left\{-\frac{G^{m+1}\pi^m(1-\gamma)^m}{\gamma^m K^m(m+1)}\exp\left(-\frac{\lambda\gamma K R^2}{m+1}\right)\right\} \quad (22)$$

which gives a clear characterization for low-coherence routing selections. As expected, multihop routing with the assistance of randomly deployed relays improves the success probability by providing large potential diversity, with the randomness in both relay locations and channel states proving helpful.

We note that unlike the single-hop scenario [10], our bound for outage probability without retransmissions is not globally monotonically increasing with λ if $D_m\nrightarrow\infty$. For sufficiently large but not infinite D_m, the outage probability can be approximated through a first-order Taylor expansion in the low-density regime

$$\begin{aligned} &p_{\rm out}^{(m)}(\lambda)\\ &\geq \exp\left\{-\frac{G\kappa^m}{m+1}\left[\exp\left(-\frac{\Lambda R^2}{m+1}\right)-\exp\left(-\Lambda D_m\right)\right]\right\}\\ &\approx 1-\lambda\frac{G\kappa^m}{m+1}\left(D_m-\frac{R^2}{m+1}\right) \end{aligned} \quad (23)$$

which indicates large outage probability in the low-density regime, arising from the difficulty in guaranteeing a relay within range $d_m(Z_m)\leq D_m$ in a sparse network. The detailed monotonicity can be more closely examined by studying the function $f(\lambda)=\exp(-a\lambda)-\exp(-b\lambda)$ ($b>a>0$), whose derivative can be computed as

$$f'(\lambda)=\exp(-b\lambda)\left\{b-a\exp\left[(b-a)\lambda\right]\right\}. \quad (24)$$

The maximum value of $f(\lambda)$ occurs at $\lambda_0=\frac{1}{b-a}\ln\frac{b}{a}$, and $f(\lambda)$ is monotonically increasing at $(0,\lambda_0]$ and decreasing at (λ_0,∞). Using this property, and defining $\Delta=\frac{R^2}{(m+1)D_m}$, we can see that

$$\min p_{\rm out}^{(m)}(\lambda)\geq\exp\left\{\frac{G\kappa^m}{m+1}\left[\Delta^{\frac{1}{1-\Delta}}-\Delta^{\frac{\Delta}{1-\Delta}}\right]\right\} \quad (25)$$

where the minimizing λ is

$$\lambda_0=\frac{1}{\gamma K\left(D_m-\frac{R^2}{m+1}\right)}\ln\frac{(m+1)D_m}{R^2}. \quad (26)$$

Hence, $p_{\rm out}^{(m)}(\lambda)$ is monotone in both $[0,\lambda_0]$ and (λ_0,∞). Taking the inverse over (λ_0,∞) will yield the bounds on maximum contention density.

C. Transmission Capacity Upper Bound

When $D_m\to\infty$, λ_0 goes to 0. Therefore, $p_{\rm out}^{(m)}(\lambda)$ is monotonically increasing in $(0,\infty)$. Therefore, we can get the following transmission capacity bound by taking the inverse of this outage probability function.

Corollary 2:

1) If each hop adopts a single transmission, the transmission capacity can be bounded as

$$\begin{aligned} T_m(\epsilon) &\leq \frac{m\ln\frac{G\pi(1-\gamma)}{K\gamma}+\ln G-\ln(m+1)-\ln\ln\frac{1}{\epsilon}}{KR^2}(1-\epsilon)\\ &\triangleq T_m^{\rm ub}(\epsilon) \end{aligned} \quad (27)$$

where $\epsilon\geq\exp\left(-\frac{G\kappa^m}{m+1}\right)$.

2) If best effort retransmissions is adopted with each hop utilizing k_i retransmissions, the transmission capacity can be bounded as

$$T_m(\epsilon)\leq\left(\sum_{i=1}^{m+1}\frac{1}{k_i}\right)(1-\epsilon)\left\{\frac{m\ln\frac{G\pi(1-\gamma)}{K\gamma}+\ln(Gm)}{KR^2\sum_{i=1}^{m+1}k_i}+\frac{-\ln\left[\left(\prod_{i=1}^{m+1}k_i\right)\left(\sum_{i=1}^{m+1}\frac{1}{k_i}\right)\right]-\ln\ln\frac{1}{\epsilon}}{KR^2\sum_{i=1}^{m+1}k_i}\right. \quad (28)$$

where $\epsilon\geq\exp\left(-\frac{mG\kappa^m}{\left(\prod_{i=1}^{m+1}k_i\right)\left(\sum_{i=1}^{m+1}\frac{1}{k_i}\right)}\right)$.

Proof: See Appendix E. ■

In the case where each hop use a single transmission, when $D_m \nrightarrow \infty$ but is reasonably large, the outage probability can be approximated using L'Hôspital's rule

$$\epsilon \geq \exp\left\{-\frac{G\kappa^m}{m+1}\left[\exp\left(-\frac{\Lambda R^2}{m+1}\right)-\exp\left(-\frac{\Lambda D_m}{m+1}\right)\right]\right\}.$$

By simple manipulation, the upper bound $T_m^{\rm ub}(\epsilon, D_m)$ on transmission capacity with D_m constraint becomes

$$T_m^{\rm ub}(\epsilon, D_m)+\Theta\{\exp\{-T_m^{\rm ub}(\epsilon, D_m)K(m+1)D_m\}\} = T_m^{\rm ub}(\epsilon) \quad (29)$$

which means the gap between the general bound and the bound with distance constraints will decay exponentially fast with D_m.

It should be noted that if we only impose a constraint on the maximum number of allowable attempts M, it is difficult to get a closed-form capacity bound. But since the outage bound is monotonically increasing with λ, it would allow a numerical solution.

IV. Numerical Analysis and Discussion

In this section, we study the implications of the theoretical results through simple numerical analysis and simulation. The presented plots presume path-loss attenuation, Rayleigh fading, and no noise. The SIR threshold β is set to 1 while other parameters are varied. The $\mathcal{S}$–$\mathcal{D}$ distance is primarily $R=4$ although $R=6$ is also used. In a interference-limited environment, the exact values of R and λ are not particularly important since the outage probability is constant for fixed λR^2. In the simulation, a spatial Poisson point process is generated. For each spatial density λ, we pick $\mathcal{S}$–$\mathcal{D}$ pairs uniformly at random that has an average spatial density $\lambda\gamma$, and let each $\mathcal{S}$–$\mathcal{D}$ pair perform uncoordinated path selection. If there is any hop conflicting with routes selected by other $\mathcal{S}$–$\mathcal{D}$ pair, then the transmission fails. The set of connected paths from $\mathcal{S}$ to $\mathcal{D}$ is determined through SIR measurements of each link, and constrained to a maximum end-to-end distance of $D_m = 60^2$ which qualitatively approximates the extreme case of $D_m \rightarrow \infty$ for $R \leq 6$.

A. Tightness of Outage Lower Bound

The lower bound (22) is plotted against simulated outage probability in Fig. 2. The simulated outage probability takes into account the dependence among consecutive transmissions and parallel path selection. For each data point, an error bar is plotted to indicate the confidence interval of the simulation results. Here, the width of the confidence interval is chosen to be twice the empirical standard deviation. The bound is observed to indeed be a lower bound and to be quite tight, albeit slightly looser for increasing numbers of hops.

B. Number of Relays m

Since $\Theta(\ln(m+1))$ is negligible compared to $\Theta(m)$, the transmission capacity bound (27) exhibits near linear scaling behavior with respect to the number of relays m. This gain arises from the increasing route diversity as m grows, since more hops allow more potentially successful routes. This gain does not depend on the noise level and is not achieved by predetermined routing approaches, which primarily are useful for overcoming per-hop range limitations (i.e., noise). We caution that this upper bound is likely to be increasingly optimistic for large m, since longer potential routes will presumably result in higher correlation between candidate paths. Fig. 3 shows the maximum allowable contention density versus the number of hops for different outage constraints. As expected, the effective contention density scales nearly linearly for small m as expected, and then diminishes rapidly for large m. In practice, a modest number of hops would be taken since longer routes experience larger delay and more protocol overhead. The proper choice of m under realistic correlation and protocol overhead models is an interesting topic for future research.

C. Outage Probability Constraint ϵ

The transmission capacity bound is not sensitive to the outage constraint ϵ in the low-outage regime, because the double logarithm as in $\ln\ln\frac{1}{\epsilon}$ largely reduces its sensitivity. For instance, when the target ϵ is decreased from 10^{-2} to 10^{-4}, the throughput only experiences a small constant loss. This is quite different than single-hop transmission capacity, which exhibits linear scaling with ϵ in the low-outage regime and so going from $\epsilon = 10^{-2}$ to $\epsilon = 10^{-4}$ would in fact decrease the transmission capacity by two orders of magnitude [10]. Hence, multihop transmission capacity is apparently much more robust to severe QoS constraints compared to single hop.

D. Availability of Relays

Recall that nodes in the network are divided into a fraction γ that may transmit and $1-\gamma$ that are available as relays. Corollary 2 implies that increasing the pool of relay nodes will logarithmically increase throughput, so the diversity gains diminish rapidly once a large enough pool to guarantee multihop route selection exists. Note that we primarily consider fixed-portion relay models here, which means the density of the pool of relays grows along with the density of source nodes. Simulations in Fig. 4 show the maximum contention density versus the intensity ratio $\frac{1-\gamma}{\gamma}$ of relays to source nodes, with $\epsilon = 0.05$. The results can be modified to study fixed-density relay models (where the density of relay nodes is a fixed constant λ_r) by substituting $\frac{1-\gamma}{\gamma}$ with λ_r/λ_t, which we do not present here.

E. Sum-Squared-Distance Constraint $\mathbf{D}_m$

The gap between the distance-constrained maximum density $T_m(\epsilon, D_m)$ and the transmission capacity $T_m(\epsilon)$ is subject to exponential decay with respect to $(m+1)D_m$ as predicted in (29). Hence, searching for multihop routes in a local region should be sufficient. Fig. 3 illustrates this when the $\mathcal{S}$–$\mathcal{D}$ distance is $R=4$. It can be observed that when D is reasonably large compared with R^2, increasing D_m provides almost no throughput gain. Also, this gain shrinks rapidly as m increases, which can also be expected from (29).

F. Limitations and Future Directions

The results of this paper are well suited to both fading and nonfading channels, but care should be exercised in considering

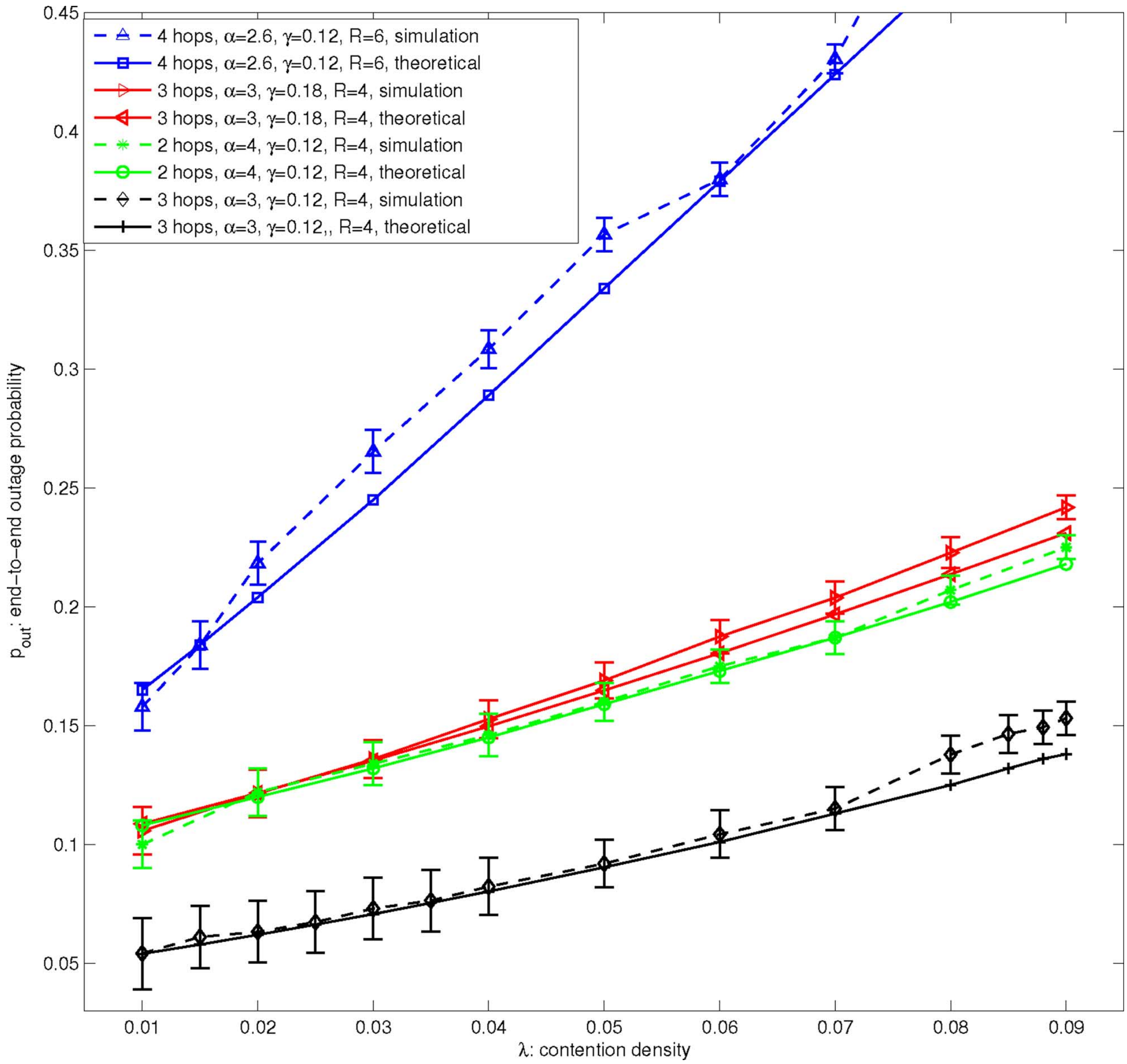

Fig. 2. End-to-end outage probability $p_{\rm out}$ computed numerically and its lower bound versus the contention density λ of all potential transmitters. On each data point on the simulation curves, an error bar indicating the confidence interval (which is chosen to be the empirical standard deviation) is plotted. Note that the lower bound is fairly tight for transmissions with two hops and three hops, and becomes looser for transmissions through four hops.

more diverse channel models like log-normal shadowing, which do not necessarily lead to an exponential outage probability expression [11]. In addition, the models in this paper assume mutual independence among different links, which would not hold in general, particularly for routing and scheduling strategies that require cooperation. Furthermore, the theoretical gap between our upper bound and the true capacity is unknown, and how it grows with the number of hops is of interest.

The multihop transmission capacity bound shows that dynamic routing selection is of significant importance when there is sufficient randomness in the network as far as path gains and interference. In fact, predetermined routing (like nearest neighbor) is unlikely to approach the throughput bound in interference-limited networks. A simple argument shows this. Considering a typical source–destination pair, the outage probability can be bounded as

$$\begin{aligned} 1-p_{\rm out}^{(m)} &= G\exp(-\lambda\gamma K d_m(Z_M)) \\ &\leq G\exp\left(-\lambda\gamma K\frac{R^2}{m+1}\right). \end{aligned} \tag{30}$$

The equality can be achieved if and only if the m relays are equally spaced along the line segment between source and destination. In fact, from the properties of Poisson random process, this is almost surely unlikely to occur, resulting in a strict inequality. Setting $\lambda\gamma(1-\epsilon)/(m+1)$ to $T_m^{\rm ub}(\epsilon)$, we can immediately get an upper bound

$$T_m^{\rm ub}(\epsilon) = \frac{1-\epsilon}{KR^2}\ln\frac{G}{1-\epsilon} \tag{31}$$

which is exactly equal to the single-hop case. This suggests that predetermined routing will not provide further throughput gain in interference-limited networks compared with single-hop direct transmission.

We note that the power-limited regime (i.e., including noise) is not considered in this paper. Although noise is unimportant in the high-density regime, it can be quite important in the low-density regime, which is often power-limited. Our framework is primarily based upon an exponential form of per-hop success probability, which does not hold in the low SNR case. From a capacity perspective, the high-density case is of more interest,

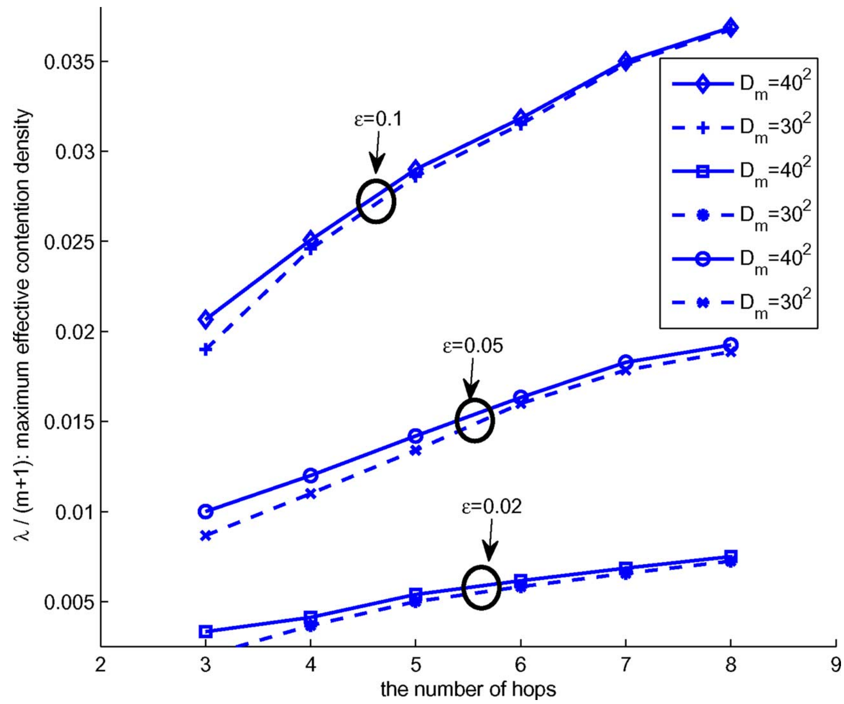

Fig. 3. Maximum allowable effective contention density $\lambda/(m+1)$ computed numerically versus the number of hops when $R = 4, \alpha = 3, \beta = 1$ for a typical $\mathcal{S}$–$\mathcal{D}$ pair. It can be seen that the maximum effective density scales nearly linearly in the number of hops. Note that increasing the distance constraint D only provides fairly small throughput gain.

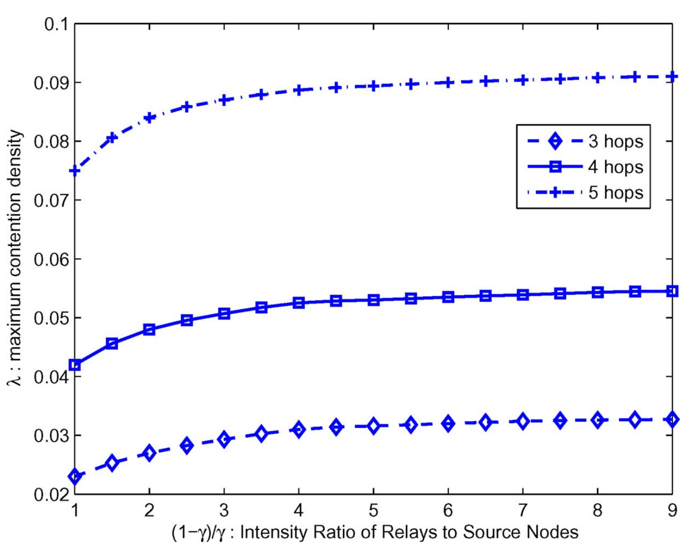

Fig. 4. Maximum allowable contention density λ computed numerically versus the intensity ratio of relays to source nodes $\frac{1-\gamma}{\gamma}$ when $\epsilon = 0.05$ for a typical $\mathcal{S}$–$\mathcal{D}$ pair. It can be observed that the maximum density scales logarithmically in $\frac{1-\gamma}{\gamma}$.

since in the low-density (power-limited) regime, nodes have far fewer options as far as selecting relays, and spatial reuse in the network is not very important. In fact, multihopping is known to be particularly helpful in changing a power-limited network to an interference-limited one by increasing the SNR in each hop, consistent with [22] and [26].

The design of transmission strategies that exploits the path diversity gain is left for future work. We conjecture that hop-by-hop route selection—which is much more realistic in a distributed network than the complete route selection assumed here—will achieve a lower diversity order (and, hence, transmission capacity).

Appendix A
Single-Hop Success Probability Under Nakagami Fading

The single-hop success probability with Nakagami fading can be developed as

$$g_0(r_{ij}, \lambda_t) = \int_0^\infty \mathbb{P}\left(\frac{z r_{ij}^{-\alpha}}{t} \geq \beta\right) f_{I_\Phi}(t)dt = \sum_{k=0}^{m_0-1} \frac{(-m_0\beta r_{ij}^\alpha)^k}{k!} \mathcal{L}_{I_\Phi}^{(k)}(m_0\beta r_{ij}^\alpha) \quad (32)$$

where $\mathcal{L}_{I_\Phi}(s)$ is the Laplace transform of the general Poisson shot noise process, and $\mathcal{L}_{I_\Phi}^{(k)}(s)$ denotes the kth derivative of $\mathcal{L}_{I_\Phi}(s)$. The closed-form formulas of them are given by [15] as

$$\mathcal{L}_{I_\Phi}(s) = \exp\left\{-\lambda_t \Omega_{m_0}\left(\frac{s}{m_0}\right)^{\frac{2}{\alpha}}\right\} \quad (33)$$

$$\mathcal{L}_{I_\Phi}^{(k)}(s) = \frac{\exp\left\{-\lambda_t\Omega_{m_0}\left(\frac{s}{m_0}\right)^{\frac{2}{\alpha}}\right\}}{(-s)^k} \sum_{j=1}^{k}\left[\frac{-2\lambda_t\Omega_{m_0}}{\alpha}\left(\frac{s}{m_0}\right)^{\frac{2}{\alpha}}\right]^j \Upsilon_{k,j} \quad (34)$$

where $\Upsilon_{k,j}$ is a constant defined in [15], and

$$\Omega_{m_0} = \frac{2\pi}{\alpha}\sum_{k=0}^{m_0-1}\binom{m}{k}B(k+\frac{2}{\alpha}, m_0-k-\frac{2}{\alpha}) \tag{35}$$

with $B(a,b)$ denoting Beta function. By manipulation, we have

$$g_0(r_{ij},\lambda_t) = \exp\left\{-\lambda_t\Omega_{m_0}\beta^{\frac{2}{\alpha}}r_{ij}^2\right\}\cdot \left\{1+\sum_{k=1}^{m_0-1}\sum_{l=1}^{k}\frac{1}{k!}\left[-\frac{2\lambda_t\Omega_{m_0}\beta^{\frac{2}{\alpha}}r_{ij}^2}{\alpha}\right]^l\Upsilon_{k,l}\right\}.$$

Generally speaking, this does not have an expected exponential form. But we can simplify the expression in certain cases. For small single-hop outage constraint ϵ, we have $\lambda_t\Omega_{m_0}\beta^{\frac{2}{\alpha}}r_{ij}^2 \ll 1$; therefore, $g_0(r_{ij},\lambda_t)$ can be approximated as

$$g_0(r_{ij},\lambda_t) \approx \exp\left\{-\lambda_t\Omega_{m_0}\beta^{\frac{2}{\alpha}}r_{ij}^2\right\}. \tag{36}$$

In contrast, for large single-hop outage regime, i.e., $\lambda_t\Omega_{m_0}\beta^{\frac{2}{\alpha}}r_{ij}^2 \gg 1$, employing L'Hospital's rule yields

$$g_0(r_{ij},\lambda_t) \approx \left\{1+\sum_{k=1}^{m_0-1}\sum_{l=1}^{k}\frac{l!}{k!}\left(-\frac{2}{\alpha}\right)^l\Upsilon_{k,l}\right\}\exp\left\{-\lambda_t\Omega_{m_0}\beta^{\frac{2}{\alpha}}r_{ij}^2\right\}.$$

We summarize them as follows

$$\text{low-outage regime: } K_{\text{NF}} = \Omega_{m_0}\beta^{\frac{2}{\alpha}},\ G_{\text{NF}} = 1$$
$$\text{high-outage regime: } K_{\text{NF}} = \Omega_{m_0}\beta^{\frac{2}{\alpha}}$$
$$G = 1+\sum_{k=1}^{m_0-1}\sum_{l=1}^{k}\frac{l!}{k!}\left(\frac{-2}{\alpha}\right)^l\Upsilon_{k,l}.$$

Since practical system typically require low outage probability, our analysis may still work to a certain extent.

APPENDIX B
PROOF OF THEOREM 1

Let the high-dimensional feasible region $\mathcal{F}$ for relay sets be the allowable range to select relays determined by different routing protocols and design parameters. Denote by $\mathcal{A}$ the event that there is *no* relay set within $\mathcal{F}$ that can successfully complete forwarding. Ignoring the edge effect, we attempt to approximately divide $\mathcal{F}$ into n disjoint hypercubes $\mathcal{F}_i(1\le i\le n)$ each of equal volume. For sufficiently large n, this approximation is exact. Let $\mathcal{A}_i(1\le i\le n)$ be the event that there exists no potential relay set within $\mathcal{F}_i$ that can complete forwarding. Since the outage event $\mathcal{A}$ occurs only when there is no potential relay set in any of the region $\mathcal{F}_i$, we have $\mathcal{A} = \bigcap_{i=1}^n \mathcal{A}_i$. Consider the hypercube $\mathcal{F}_i$ as $[(x_1,y_1,\ldots,x_m,y_m), (x_1+\delta x_1, y_1+\delta y_1,\ldots,x_m+\delta x_m, y_m+\delta y_m)]$ when $\delta x_i\to 0$ and $\delta y_i \to 0$. Define $Z_i = (x_1,y_1,\ldots,x_m,y_m)$. Since this is a simple point process, we can approximate the void probability as follows if the Lebesgue measure $v_{2m}(\mathcal{F}_i)$ is small or n is sufficiently large

$$\begin{aligned}\lim_{\delta x_i\to 0,\delta y_i\to 0}\mathbb{P}(\mathcal{A}_i) &= \lim_{\delta x_i\to 0,\delta y_i\to 0} 1-g_m(Z_i,\lambda_t)\prod_{i=1}^{m}\left(1-\exp(-\tilde{\lambda}\delta x_i\delta y_i)\right) \qquad (37)\\ &= \lim_{\delta x_i\to 0,\delta y_i\to 0} 1-g_m(Z_i,\lambda_t)\prod_{i=1}^{m}\tilde{\lambda}\delta x_i\delta y_i\\ &= \lim_{\delta x_i\to 0,\delta y_i\to 0}\exp\left(-\tilde{\lambda}^m g_m(Z_i,\lambda_t)v_{2m}(\mathcal{F}_i)\right). \qquad (38)\end{aligned}$$

Consider two realizations ω and ω' of this higher dimensional point process, and denote $\omega \preceq \omega'$ if ω' can be obtained from ω by adding points. An event $\mathcal{A}_i$ is said to be increasing if for every $\omega\preceq\omega'$, $\mathbb{I}_{\mathcal{A}_i}(\omega) \le \mathbb{I}_{\mathcal{A}_i}(\omega')$ with $\mathbb{I}_{\mathcal{A}_i}$ denoting the indicator function of the event $\mathcal{A}_i$. If $\mathcal{A}_i(1\le i\le n)$ are all increasing events, then the Harris-FKG inequality [33] yields

$$\mathbb{P}(\mathcal{A}) = \mathbb{P}(\bigcap_{i=1}^n \mathcal{A}_i) \ge \prod_{i=1}^n \mathbb{P}(\mathcal{A}_i). \tag{39}$$

Letting n go to infinity, we can get the lower bound of outage probability as follows:

$$\begin{aligned}\mathbb{P}(\mathcal{A}) &\ge \lim_{n\to\infty}\prod_{i=1}^n\mathbb{P}(\mathcal{A}_i)\\ &= \exp\left(-\tilde{\lambda}^m g_m(Z_i,\lambda_t)\lim_{n\to\infty}\sum_{i=1}^n v_{2m}(\mathcal{F}_i)\right)\\ &= \exp(-\mathbb{E}(N_m)). \qquad (40)\end{aligned}$$

APPENDIX C
PROOF OF LEMMA 1

The isosurface of $d_m(Z_m) = a$ has the following coordinate geometry form:

$$X_{\text{sum}} + Y_{\text{sum}} = a \tag{41}$$

where

$$X_{\text{sum}} = \left(x_1+\frac{R}{2}\right)^2 + \sum_{i=1}^{m-1}(x_{i+1}-x_i)^2 + \left(x_m - \frac{R}{2}\right)^2$$
$$Y_{\text{sum}} = y_1^2 + \sum_{i=1}^{m-1}(y_{i+1}-y_i)^2 + y_m^2.$$

If we treat x_i, y_i $(1\le i\le m)$ as mutually orthogonal coordinates, then (41) forms a quadratic surface in $2m$-dimensional space. See Fig. 2 for an illustration when $m=1$. From the properties of quadratic forms, the x part and y part of (41) can be expressed as

$$\begin{cases} X_{\text{sum}} = (\mathbf{CX}-\mathbf{R_x})^{\mathbf{T}}\mathbf{\Lambda_x}(\mathbf{CX}-\mathbf{R_x}) + t_m R^2 \\ Y_{\text{sum}} = (\tilde{\mathbf{C}}\mathbf{Y})^{\mathbf{T}}\mathbf{\Lambda_y}(\tilde{\mathbf{C}}\mathbf{Y}) \end{cases} \tag{42}$$

where $\mathbf{C}, \tilde{\mathbf{C}}$ are orthogonal matrices, $\boldsymbol{\Lambda}_{\mathbf{x}}, \boldsymbol{\Lambda}_{\mathbf{y}}$ are diagonal matrices, $\mathbf{R}_{\mathbf{x}}$ is a m-dimensional vector, and t_m is a constant that will be determined in the sequel. Here, the orthogonal transformation of $\mathbf{X}$ ($\mathbf{Y}$) by $\mathbf{C}$ ($\tilde{\mathbf{C}}$) and translation transformation by $\mathbf{R}_{\mathbf{x}}$ only result in rotation, flipping, or translation of the quadratic surface without changing the shape of it. Since the corresponding quadratic terms of X_{sum} and Y_{sum} have equivalent coefficients, we have $\boldsymbol{\Lambda}_m \overset{\triangle}{=} \boldsymbol{\Lambda}_{\mathbf{x}} = \boldsymbol{\Lambda}_{\mathbf{y}}$. Denote the symmetric quadratic-form matrix corresponding to Y_{sum} as $\mathbf{A}_m$, then $\mathbf{A}_m$ is the following tridiagonal matrix of dimension m:

$$\mathbf{A}_m = \begin{pmatrix} 2 & -1 & 0 & \dots & 0 \\ -1 & 2 & -1 & \dots & 0 \\ 0 & -1 & 2 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 2 \end{pmatrix}. \tag{43}$$

In fact, $\boldsymbol{\Lambda}_m$ is the canonical form of $\mathbf{A}_m$ with its eigenvalues on the main diagonal. Through orthogonal transformation and translation, $X_{\text{sum}}, Y_{\text{sum}}$ can be brought to the explicit form

$$\begin{cases} X_{\text{sum}} = \sum_{i=1}^{m} \lambda_i \tilde{x_i}^2 + t_m R^2 \\ Y_{\text{sum}} = \sum_{i=1}^{m} \lambda_i \tilde{y_i}^2 \end{cases} \tag{44}$$

where $\tilde{x_i}, \tilde{y_i}$ are the new orthogonal coordinates and λ_i is the ith eigenvalue of $\mathbf{A}_m$. By its definition, X_{sum} is positive definite, and the following minimum value can be obtained if and only if m relays are placed equidistant along the line segment between the source and destination

$$\begin{aligned} X_{\text{sum}} &\geq \frac{(|\frac{R}{2} + x_1| + |x_2 - x_1| + \cdots + |x_m - \frac{R}{2}|)^2}{m+1} \\ &\geq \frac{R^2}{m+1}. \end{aligned} \tag{45}$$

Therefore, $t_m = \frac{1}{m+1}$. Now, (41) can be brought to

$$\sum_{i=1}^{m} \lambda_i \tilde{x_i}^2 + \sum_{i=1}^{m} \lambda_i \tilde{y_i}^2 = a - \frac{R^2}{m+1}. \tag{46}$$

From the positive definiteness of $\mathbf{A}_m$, $\lambda_{\mathbf{i}} > 0$ holds for all i, i.e., the aforementioned equation forms the surface of a $2m$-dimensional ellipsoid. Fig. 5 illustrates the ellipsoid when $m = 1$, which reduces to a circle. The Lebesgue measure of the ellipsoid can be written as

$$V_m(a) = \frac{\pi^m (a - \frac{R^2}{m+1})^m}{m! \prod_{i=1}^{m} \lambda_i} = \frac{\pi^m (a - \frac{R^2}{m+1})^m}{m! \det(\mathbf{A}_m)}. \tag{47}$$

We also need to determine $\det(\mathbf{A}_m)$, which can be computed by the Laplace expansion of the determinant

$$\det(\mathbf{A}_m) = 2\det(\mathbf{A}_{m-1}) - \det(\mathbf{A}_{m-2}). \tag{48}$$

Solving this recursive form with the initial value $\det(\mathbf{A}_1) = 2$ and $\det(\mathbf{A}_2) = 3$ yields

$$\det(\mathbf{A}_m) = m + 1 \Rightarrow V_m(a) = \frac{\pi^m (a - \frac{R^2}{m+1})^m}{(m+1)!}. \tag{49}$$

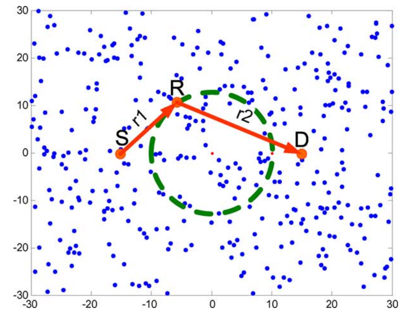

Fig. 5. In single relay scenario, the contour of end-to-end success probability $g_1 = G^2 \exp(-\lambda_t K (r_1^2 + r_2^2))$ is exactly a circle. The plot is a realization of Poisson point process of $\lambda = 0.1/\text{unit area}$. The source S and destination D are a distance 30 apart with relay R on the dotted circle satisfying $r_1^2 + r_2^2 = 500$.

Now, we can compute the outage probability. Integrating over different isosurfaces with $g(Z_m, \lambda\gamma) = \exp(-\lambda\gamma K a)$, and defining $H = \frac{\pi(1-\gamma)}{\gamma K}$, we can compute the average number of potential relay sets as

$$\begin{aligned} &\mathbb{E}(N_m) \\ &= \int_{\frac{R^2}{m+1}}^{D_m} \lambda^m (1-\gamma)^m \frac{\mathrm{d}V_m(a)}{\mathrm{d}a} G^{m+1} \exp(-\Lambda a) da \qquad (50) \\ &= G^{m+1} \lambda^m (1-\gamma)^m \cdot \\ &\quad \int_{\frac{R^2}{m+1}}^{D_m} \frac{m\pi^m (a - \frac{R^2}{m+1})^{m-1}}{(m+1)!} \exp(-\Lambda a) da \qquad (51) \\ &= \frac{mG^{m+1} H^m \exp(-\frac{\Lambda R^2}{m+1})}{(m+1)!} \int_0^{\Lambda(D_m - \frac{R^2}{m+1})} x^{m-1} e^{-x} dx \\ &= \frac{mG^{m+1} H^m e^{-\frac{\Lambda R^2}{m+1}}}{(m+1)!} \left\{ e^{-x} \sum_0^{m-1} \frac{(m-1)!}{i!} x^i \right\}_{\Lambda(D_m - \frac{R^2}{m+1})}^{0} \\ &= \frac{G^{m+1} H^m}{m+1} \left\{ \exp(-\frac{\Lambda R^2}{m+1}) - \right. \\ &\quad \left. \exp(-\Lambda D_m) \sum_0^{m-1} \frac{1}{i!} \left(\Lambda (D_m - \frac{R^2}{m+1}) \right)^i \right\}. \qquad (52) \end{aligned}$$

It is worth noting that a relay set may contain the same location for different relays. This can be interpreted as employing the same node in different frequency bands for forwarding, although this is not common in practical routing. We notice that these sets form $\binom{m}{2}$ hyperplanes in the $2m$-dimensional hyperspace, which are of measure 0. Hence, even if we require distinct relays and take the integral over feasible regions, we will still get the same results.

Appendix D
Proof of Lemma 2

We proceed in a similar spirit as in the proof of Lemma 1. Define $\mathbf{n} = (n_1, \dots, n_{m+1})^T$ and $\mathbf{k} = (k_1, \dots, k_{m+1})^T$. When the ith hop is of distance r_i and k_i attempts are employed in the ith hop, the probability for successful reception is given by

$$\begin{aligned} &g(r_1, k_1, \ldots, r_{m+1}, k_{m+1}) \\ &= \prod_{i=1}^{m+1} \left[1 - \left(1 - G\exp\left(-\lambda_t K r_i^2\right)\right)^{k_i}\right] \\ &= \prod_{i=1}^{m+1} \left[\sum_{n_i=1}^{k_i} (-1)^{n_i+1} \binom{k_i}{n_i} G^{n_i} \exp\left(-\lambda_t K n_i r_i^2\right)\right] \\ &= \sum_{\mathbf{n}:\mathbf{1}\preceq\mathbf{n}\preceq\mathbf{k}} (-1)^{m+1+\sum_{i=1}^{m+1} n_i} G^{\sum_{i=1}^{m+1} n_i} \\ &\qquad \exp\left(-\lambda_t K \sum_{i=1}^{m+1} n_i r_i^2\right) \prod_{i=1}^{m+1} \binom{k_i}{n_i} \end{aligned} \tag{53}$$

where $\mathbf{1} := (1, \ldots, 1)^T$. Therefore, we redefine $X_{\text{sum}}, Y_{\text{sum}}$ to be

$$\begin{aligned} X_{\text{sum}}^* &= n_1\left(x_1 + \frac{R}{2}\right)^2 + \sum_{i=1}^{m-1} n_{i+1}(x_{i+1} - x_i)^2 \\ &\quad + n_{m+1}\left(x_m - \frac{R}{2}\right)^2 \\ Y_{\text{sum}}^* &= n_1 y_1^2 + \sum_{i=1}^{m-1} n_{i+1}(y_{i+1} - y_i)^2 + n_{m+1} y_m^2. \end{aligned}$$

The Cauchy–Schwartz inequality indicates

$$\begin{aligned} &\left(\sum_{i=1}^{m+1} n_i r_i^2\right)\left(\sum_{i=1}^{m+1} \frac{1}{n_i}\right) \geq \left(\sum_{i=1}^{m+1} r_i\right)^2 = R^2 \\ &\qquad \Rightarrow X_{\text{sum}}^* \geq \frac{R^2}{\sum_{i=1}^{m+1} \frac{1}{n_i}}. \end{aligned} \tag{54}$$

Therefore, the Lebesgue measure of the ellipsoid $d_m^*(Z_m) \leq a$ can be calculated as

$$V_m(a)^* = \frac{\pi^m \left(a - \frac{R^2}{\sum_{i=1}^{m+1} 1/n_i}\right)^m}{m! \det\left(\mathbf{A}_m^*\right)} \tag{55}$$

where $\mathbf{A}_m^*$ is the canonical form corresponding to Y_{sum}^* and can be written as

$$\mathbf{A}_m^* = \begin{pmatrix} n_1 + n_2 & -n_2 & 0 & \ldots & 0 \\ -n_2 & n_2 + n_3 & -n_3 & \ldots & 0 \\ 0 & -n_3 & n_3 + n_4 & \ldots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \ldots & n_m + n_{m+1} \end{pmatrix}.$$

By Laplace expansion of the determinant, we get

$$\begin{aligned} \det(\mathbf{A}_m^*) &= (n_m + n_{m+1}) \det(\mathbf{A}_{m-1}^*) - n_m^2 \det(\mathbf{A}_{m-2}^*) \\ &\Rightarrow \det(\mathbf{A}_m^*) = \left(\prod_{i=1}^{m+1} n_i\right)\left(\sum_{i=1}^{m+1} \frac{1}{n_i}\right) \end{aligned} \tag{56}$$

which follows by induction. Define $a = \sum_{i=1}^{m+1} n_i r_i^2$. Taking an integral over different isosurfaces yields

$$\begin{aligned} h_{\mathbf{n}} &\triangleq \int_{\frac{R^2}{\sum_{i=1}^{m+1} 1/n_i}}^{\infty} \frac{dV_m^*(a)}{da} \exp(-\Lambda a)\, da \\ &= \int_{\frac{R^2}{\sum_{i=1}^{m+1} 1/n_i}}^{\infty} \frac{\pi^m \left(a - \frac{R^2}{\sum_{i=1}^{m+1} 1/n_i}\right)^{m-1}}{(m-1)! \left(\prod_{i=1}^{m+1} n_i\right)\left(\sum_{i=1}^{m+1} \frac{1}{n_i}\right)} e^{-\Lambda a} da \\ &= \frac{\pi^m e^{-\frac{\Lambda R^2}{\sum_{i=1}^{m+1} 1/k_i}} \int_0^\infty x^{m-1} e^{-x} dx}{\Lambda^m (m-1)! \left(\prod_{i=1}^{m+1} n_i\right)\left(\sum_{i=1}^{m+1} \frac{1}{n_i}\right)} \\ &= \frac{\pi^m m \exp\left(-\frac{\Lambda R^2}{\sum_{i=1}^{m+1} 1/k_i}\right)}{\Lambda^m \left(\prod_{i=1}^{m+1} n_i\right)\left(\sum_{i=1}^{m+1} \frac{1}{n_i}\right)}. \end{aligned} \tag{57}$$

By combining (53) and (57), we can derive the average number of relay sets when retransmitting k_i times in the ith hop as

$$\begin{aligned} &\mathbb{E}_{\mathbf{k}}(N_m) \\ &= \lambda^m (1-\gamma)^m \sum_{\mathbf{n}:\mathbf{1}\preceq\mathbf{n}\preceq\mathbf{k}} (-1)^{m+1} \cdot \\ &\qquad (-G)^{\sum_{j=1}^{m+1} n_j} \prod_{i=1}^{m+1} \binom{k_i}{n_i} h_{\mathbf{n}} \\ &= \lambda^m (1-\gamma)^m \sum_{\mathbf{n}:\mathbf{1}\preceq\mathbf{n}\preceq\mathbf{k}} (-1)^{m+1} (-G)^{\sum_{j=1}^{m+1} n_j} \cdot \\ &\qquad \prod_{i=1}^{m+1} \binom{k_i}{n_i} \frac{\pi^m m \exp\left(-\frac{\Lambda R^2}{\sum_{j=1}^{m+1} 1/k_j}\right)}{\Lambda^m \left(\prod_{i=1}^{m+1} n_i\right)\left(\sum_{i=1}^{m+1} \frac{1}{n_i}\right)} \\ &= \frac{(-1)^{m+1} \pi^m (1-\gamma)^m}{\gamma^m K^m} \sum_{\mathbf{n}:\mathbf{1}\preceq\mathbf{n}\preceq\mathbf{k}} (-G)^{\sum_{j=1}^{m+1} n_j} \cdot \\ &\qquad \prod_{i=1}^{m+1} \binom{k_i}{n_i} \frac{m \exp\left(-\frac{\Lambda R^2}{\sum_{j=1}^{m+1} 1/k_j}\right)}{\left(\prod_{i=1}^{m+1} n_i\right)\left(\sum_{i=1}^{m+1} \frac{1}{n_i}\right)}. \end{aligned} \tag{58}$$

In addition, we can impose a constraint on the maximum total number of attempts M without specifying the number of transmissions for each hop. For a typical relay set with the ith hop of distance r_i, we denote by p_i the success probability of hop i in any time slot. Among these M time slots, successful reception occurs when there exists $m+1$ slots $t_i (1 \leq i \leq m+1)$ that satisfy: 1) transmission in the ith hop is successful at time t_i; 2) for $1 \leq i < j \leq m+1$, we have $1 \leq t_i < t_j \leq M$. We apply a greedy approach to search for all possible scenarios that allow successful reception, which can be determined by the smallest $t = (t_1, \ldots, t_{m+1})$ that satisfies the aforementioned two requirements. By "smallest" we mean there is no $\tilde{t} \preceq t$

that meets the requirement. This is identical to finding the interval $(t_1-1, t_2-t_1-1, \ldots, t_{m+1}-t_m-1)$, or equivalently, finding a vector $\mathbf{j}=(j_1,\ldots,j_{m+1})$ such that $\mathbf{j}\succeq 0$ and $\mathbf{j}^T\cdot\mathbf{1}\leq M-m-1$. Hence, the success probability with $m+1$ hop routing can be calculated as

$$\begin{aligned} &g_{(m,M)}(r_1,\ldots,r_{m+1}) \\ &= \prod_{i=1}^{m+1} p_i \left\{ \sum_{\substack{\mathbf{j}^T\cdot\mathbf{1}\leq M-m-1 \\ \mathbf{j}\succeq 0}} \prod_{i=1}^{m+1}(1-p_i)^{j_i} \right\} \qquad (59) \\ &= \left(\prod_{i=1}^{m+1} p_i\right) \sum_{\substack{\mathbf{j}^T\cdot\mathbf{1}\leq M-m-1 \\ \mathbf{j}\succeq 0}} \sum_{0\preceq\mathbf{k}\preceq\mathbf{j}} (-1)^{\mathbf{k}^T\cdot\mathbf{1}} \prod_{l=1}^{m+1}\binom{j_l}{k_l}\prod_{i=1}^{m+1} p_i^{k_i} \\ &= \sum_{\substack{\mathbf{k}^T\cdot\mathbf{1}\leq M-m-1 \\ \mathbf{k}\succeq 0}} (-1)^{\mathbf{k}^T\cdot\mathbf{1}} \sum_{\substack{\mathbf{j}\succeq\mathbf{k} \\ \mathbf{j}^T\cdot\mathbf{1}\leq M-m-1}} \prod_{l=1}^{m+1}\binom{j_l}{k_l}\prod_{i=1}^{m+1} p_i^{k_i+1} \\ &= \sum_{\substack{\mathbf{k}^T\cdot\mathbf{1}\leq M \\ \mathbf{k}\succeq\mathbf{1}}} (-1)^{\mathbf{k}^T\cdot\mathbf{1}} \sum_{\substack{\mathbf{j}^T\cdot\mathbf{1}\leq M \\ \mathbf{j}\succeq\mathbf{k}}} \prod_{l=1}^{m+1}\binom{j_l-1}{k_l-1}\prod_{i=1}^{m+1} p_i^{k_i}. \qquad (60) \end{aligned}$$

Therefore, we can obtain

$$\mathbb{E}_M(N_m) = \sum_{\substack{\mathbf{k}^T\cdot\mathbf{1}\leq M \\ \mathbf{k}\succeq\mathbf{1}}} (-1)^{\mathbf{k}^T\cdot\mathbf{1}} \sum_{\substack{\mathbf{j}^T\cdot\mathbf{1}\leq M \\ \mathbf{j}\succeq\mathbf{k}}} \prod_{l=1}^{m+1}\binom{j_l-1}{k_l-1}\mathbb{E}_{\mathbf{k}}(N_m)$$

where $\mathbb{E}_{\mathbf{k}}(N_m)$ is given in (58).

Appendix E
Proof of Corollary 2

Let $D_m \to \infty$, then the integral part in (52) becomes a Gamma function

$$\lim_{D_m\to\infty}\int_0^{\lambda\gamma K(D_m-\frac{R^2}{m+1})} x^{m-1}e^{-x}\mathrm{d}x = \Gamma(m) = (m-1).$$

By setting the outage probability to be ϵ, we can simplify (19) as

$$\epsilon \geq \exp\left\{-\frac{G^{m+1}\pi^m(1-\gamma)^m}{\gamma^m K^m(m+1)}\exp\left(-\frac{\lambda\gamma}{m+1}KR^2\right)\right\}. \quad (61)$$

Notice that the effective spatial density is $\lambda\gamma/(m+1)$ and that ϵ is monotonically increasing with respect to λ, we can immediately derive

$$\frac{\lambda}{m+1} \leq \frac{m\ln\frac{G\pi(1-\gamma)}{K\gamma} + \ln G - \ln(m+1) - \ln\ln\frac{1}{\epsilon}}{KR^2} \quad (62)$$

which yields (27). Furthermore, in order to make the capacity well defined, i.e., $T_m(\epsilon) > 0$, we will have the constraint for outage probability stated in the corollary.

The derivation in a best effort setting is exactly the same.

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